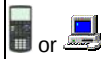
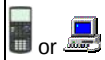
Unit 6: Financial Applications of Exponential Functions (8 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

- Connecting compound interest to exponential growth
- Examining annuities using technology
- Making decisions and comparisons using the TVM solver

DAY	Lesson Title & Description	2P	2D	Expectations		Teaching / Assessment Notes and Curriculum Sample Problems
1	Interested in Your Money • Investigating and defining financial terminology • Calculating and comparing simple and compound interest ***Lesson Included***	N	N	EF3.01 ✓	compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time	 ***Sample problem:*** Compare, using tables of values and graphs, the amounts after each of the first five years for a $1000 investment at 5% simple interest per annum and a $1000 investment at 5% interest per annum, compounded annually.
2	Connecting Compound Interest & Exponential Growth • Connecting simple interest with linear growth • Connecting compound interest with exponential growth ***Lesson Included***	N	N	EF3.01 ✓		
		N	N	EF3.03 ✓	determine, through investigation (e.g., using spreadsheets and graphs), that compound interest is an example of exponential growth [e.g., the formulas for compound interest, $A = P(1 + i)^n$, and present value, $PV = A(1 + i)^{-n}$, are exponential functions, where the number of compounding periods, n, varies]	***Sample problem:*** Describe an investment that could be represented by the function $f(x) = 500(1.01)^x$.
3	There's Gotta Be a Faster Way • Developing the compound interest formula • Solving problems using the compound interest formula	N	N	EF3.03 ✓		*Use homework from Day 2 as a rationale for finding a formula to calculate compound interest. Make connections between constant ratio in the table from this homework and the (1+i) in the formula.*
		N	N	EF3.02	solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), and the principal, P (also referred to as present value, PV), using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV=PV(1+i)^n$]	***Sample problem:*** Calculate the amount if $1000 is invested for three years at 6% per annum, compounded quarterly.

4	TVM Solver • Introduction on how to use the TVM solver for compound interest • Using TVM solver to calculate time and interest rates	N	N	EF3.04	solve problems, using a TVM Solver in a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period, i, or the number of compounding periods, n, in the compound interest formula $A = P(1 + i)^n$ [or $FV=PV(1+i)^n$]	 *Note: To introduce the TVM solver to students, use the TVM solver to check answers to previous day's work.* ***Sample problem:*** Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.
5	Annuities • Defining the term annuity • Investigate the amount of an annuity using technology	N N	N N	EF3.05 ✓ EF3.07 ☑	explain the meaning of the term *annuity*, through investigation of numerical and graphical representations using technology; solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity in situations where the compounding period and the payment period are the same (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).	 *Note: it may be helpful to explore annuities on a spreadsheet first so that students can "see" how the investment is growing.* *Note: it is not an expectation to develop or use an annuity formula. Students should always use a spreadsheet or a TVM solver.*
6	Saving • Exploration of annuities involving earning interest • Examining total interest	N	N	EF3.07 ☑		
7	Borrowing • Exploration of annuities involving paying interest (loans) • Examining total interest					

8	Changing Conditions • Examine annuity scenarios where conditions are changed (payments, interest rate, etc) and make conclusions about the effects. ***Lesson Included***	N	N	EF3.06 ☑	determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities in situations where the compounding period and the payment period are the same (e.g., long-term savings plans, loans)	 ***Sample problem:*** Compare the amounts at age 65 that would result from making an annual deposit of $1000 starting at age 20, or from making an annual deposit of $3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?);
9	Review Day (Jazz Day)					
10	Summative Unit Evaluation					

Unit 6 : Day 1 : "Interest"ed in Your Money

Grade 11 U/C

Minds On: 20

Action: 40

Consolidate:15

Total=75 min

Description/Learning Goals

- Introduce and define common financial terminology.
- Compare, using a table of values, the simple and compound interest earned for a given principal and a fixed interest rate over time.

Materials

- Money tray
- Newspapers
- Calculators
- BLM 6.1.1
- BLM 6.1.2
- BLM 6.1.3

Assessment Opportunities

Minds On…

Pairs → Exploration

Students will work in pairs to search for interest rates in a newspaper to see where in everyday life these are used. (Examples would be bank rates, mortgage rates, car loans, etc.) List the various interest rates that students find in the media and use them as part of a review of converting percents to decimals.

If newspapers are not readily available, teachers can use bank websites to get the appropriate rates.

Individual → Practice

Students will complete BLM 6.1.1.

Action!

Whole Class → Teacher-led Investigation

Define "interest", "simple interest", "compound interest", "principal", "rate", "balance", "term" and "per annum". Discuss with students how to calculate simple interest on an amount.

Students will investigate the differences between simple and compound interest in a demonstration. Give 2 students $1000 and assign them a bank. One bank pays simple interest while the other pays compound interest. An interest rate of 5% can be used. The teacher will be the "bank manager" in each case and distribute the interest to the 2 students. All students should be given BLM 6.1.2 to record the amount of money each "investor" has at the end of each year.

Teachers may wish to develop a "word wall" for the financial terminology. Additional information on the word wall can be found in *Think Literacy. Cross-curricular Approaches Grades 7-12. 2003. pg. 12-14.*

Teachers will need to develop the simple interest formula but should not develop the formula for compound interest at this time.

Consolidate Debrief

Whole Class → Investigation

Students compare the totals of each student to deduce that compound interest earns more interest than simple interest.

Ask students if they think this will happen for all amounts and interest rates.

Home Activity or Further Classroom Consolidation

Have students practice calculating simple and compound interest using different principal amounts and different annual interest rates by completing BLM 6.1.3.

Concept Practice
Skill Drill

Mathematical Process Focus: Selecting Appropriate Tools and Strategies (Students will need to choose the appropriate strategy to use depending on whether the problem involves simple or compound interest)

6.1.1 Percents to Decimals

The word "percent" means "out of 100". To convert a percent to a decimal, simply divide the percent by 100, or move the decimal place two spaces to the left and remove the percent sign.

Example: Convert 64% to a decimal.

$$64\% = \frac{64}{100} = 0.64$$

or

$$64\% = 0.\underbrace{64}_{2\text{ places}}$$

1. Convert each percent to a decimal.

a) 7%

b) $6\frac{1}{4}\%$

c) 0.5%

d) 0.008%

Example: Find 12% of 150.

$$12\% \times 150$$
$$= 0.12 \times 150$$
$$= 18$$

Hint: Recall that "of" means to multiply.

2. Find each value.

a) 7% of \$250

b) 3.5% of \$127.79

3. Find the total cost including sales tax (GST and PST) on a compact disc that costs \$16.99.

6.1.2 An Interesting Problem

Write an explanation of simple interest in the space below.

Simple Interest: _____

_____

Interest for year 1: $I = Prt$

$I = (\$1000)(0.05)(1)$

$I = \$50$

Complete the chart below for the simple interest bank.

Simple Interest Bank Chart

Year	Starting Balance	Balance that Interest is Calculated On	Interest	Ending Balance
1	$1000	$1000	I = (1000)(.05)(1) = $50	$1050
2	$1050	$1000	I = (1000)(.05)(1) = $50	
3				
4				
5				

Calculations: Use the space below for any calculations needed to help you fill in the rest of the chart.

6.1.2 An Interesting Problem (continued)

Write an explanation of compound interest in the space below.

Compound Interest: _____

_____

Complete the chart below for the simple interest bank.

Compound Interest Bank Chart

Year	Starting Balance	Balance that Interest is Calculated On	Interest (I = P*rt*)	Ending Balance
1	$1000	$1000	I = (1000)(.05)(1) = $50	$1050
2	$1050	$1050	I = (1050)(.05)(1) =	
3				
4				
5				

Calculations: Use the space below for any calculations needed to help you fill in the rest of the chart.

6.1.3 Simple and Compound Interest

1. You invest $250 at 4% per annum at a bank that pays simple interest.

   a) How much simple interest would be earned each year?

   b) If you kept your money invested for 8 years, how much total simple interest would be earned?

   c) How much money would be in your bank account after the 8 years if you did not withdraw any money?

2. If you doubled the principal from question 1, would it double the total interest paid over 8 years?

3. If you invested at double the interest rate from question 1, would it double the total interest paid over 8 years?

4. You invest $750 at 6% per annum at a bank that pays compound interest.

   a) How much compound interest would be earned in the first year?

   b) How much **more** compound interest would be earned in the second year?

   c) If you kept your money invested for 8 years, how much total compound interest would be earned? (You may want to complete a chart similar to the one on page 2 of 6.1.2.

   d) How much money would be in your bank account after the 8 years if you did not withdraw any money?

5. If you doubled the principal from question 2, would it double the total interest paid over 8 years?

6. If you invested at double the interest rate from question 1, would it double the total interest paid over 8 years?

7. Some items (such as antiques, rare stamps or land value) you purchase will increase in value over time. This is called appreciation.

   Yesterday you bought a 1913 gold King George V $5 coin for $200. If the coin appreciates by 4% per year, how much will it be worth at the end of 5 years?

Unit 6 : Day 2 : Connecting Compound Interest & Exponential Growth

Grade 11 U/C

Minds On: 20

Action: 30

Consolidate: 25

Total=75 min

Description/Learning Goals

- Students will relate simple interest to linear growth and compound interest to exponential growth.
- Students will be able to convert an interest rate to a constant ratio.

Materials

- BLM 6.2.1
- BLM 6.2.2

Assessment Opportunities

Minds On…

Whole Class → Review

Take up 6.1.3 homework. Review the simple interest formula and the method that was used for completing the compound interest table.

Action!

Whole Class → Guided Exploration

Work through BLM 6.2.1 with the class. Students will have the opportunity to make connections between simple interest and linear growth and compound interest and exponential growth by examining first differences and graphs.

It is hoped that students will look at the first differences in the charts and make some conclusions about the types of relationships.

Students may need to be review the definition of constant ratio.

Mathematical Process: Connecting (students will make the **connections** between simple interest and linear growth; and between compound interest and exponential growth.)

Consolidate Debrief

Whole Class → Reflection

Teachers should review the "big ideas" from this section:
Compound interest grows exponentially; the interest rate is converted to the constant ratio by converting the rate to a decimal and then adding one.

Students can begin BLM 6.2.2.

Home Activity or Further Classroom Consolidation

Students are to complete BLM 6.2.2 for homework.

Application
Concept Practice
Exploration

6.2.1 Simple vs Compound Interest: What's the Difference?

Recall the Simple and Compound Interest Tables from yesterday:

Simple Interest

Year	Starting Balance	Balance that Interest is Calculated on	Interest	Ending Balance
1	$1000	$1000	$50	$1050
2	$1050	$1000	$50	$1100
3	$1100	$1000	$50	$1150
4	$1150	$1000	$50	$1200
5	$1200	$1000	$50	$1250

1. What type of relationship exists between "year" and "ending balance"? Give a reason for your answer.

Compound Interest

Year	Starting Balance	Balance that Interest is Calculated on	Interest	Ending Balance
1	$1000	$1000	$50	$1050
2	$1050	$1050	$52.50	$1102.50
3	$1102.50	$1102.50	$55.13	$1157.63
4	$1157.63	$1157.63	$57.88	$1215.51
5	$1215.51	$1215.51	$60.78	$1276.29

2. What type of relationship exists between "year" and "ending balance"? Give a reason for your answer.

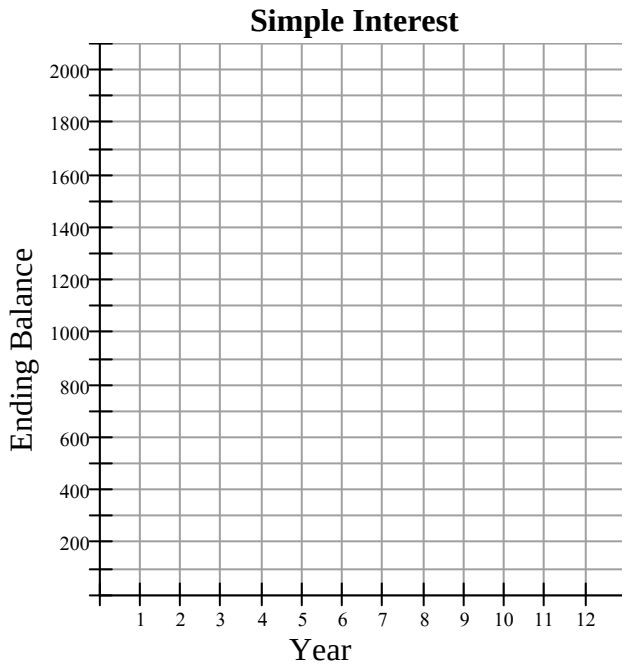
Summary:

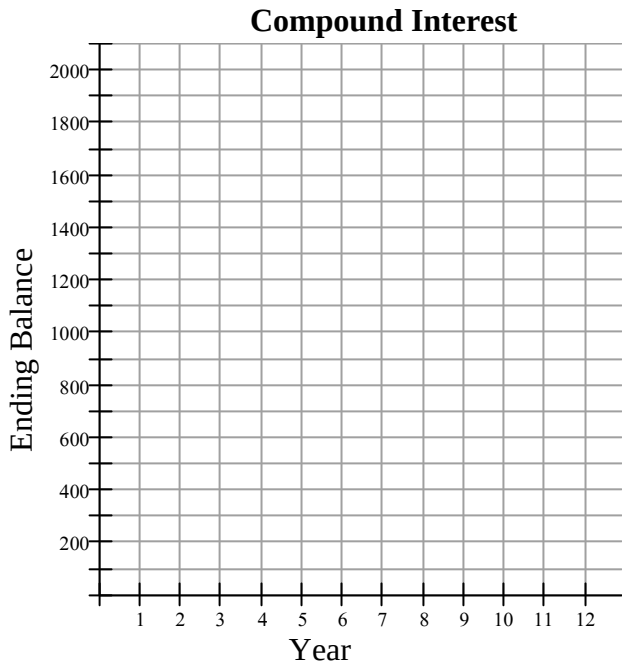
For Simple Interest, the relationship between time and the ending balance is: _____

For Compound Interest, the relationship between time and the ending balance is: _____

6.2.1 Simple vs Compound Interest: What's the Difference?
(continued)

3. (a) On the grids below, graph the relationship between "year" and "ending balance" for each of the charts.





(b) Use the graphs to determine the ending balance for each account after 10 years.

4. (a) What is the value of the constant ratio for the Compound Interest example?

(b) What seems to be the relationship between the constant ratio and the compound interest rate?

5. If a bank advertises a savings account interest rate of 6%, what would the constant ratio be?

6.2.2 Compound Interest

1. The table below shows the interest rate that four banks pay on a savings account. For each bank, determine the constant ratio.

Bank	Interest Rate	Constant Ratio
Money is our Middle Name!	6%	
Save Your Money	5.4%	
The Cash Counters	5.755	

2. Jacob sees the following ad in the newspaper:

Jacob's current bank pays him 3.2% compound interest. If he moves his money over to the ***Money Makers Investment Firm***, what will his new constant ratio be?

Money Makers Investment Firm

We Pay ½% above any bank's compound interest rate!

Call 1-555-555-EARN

3. A 100th Anniversary Harley-Davidson motorcycle ***appreciates*** (increases in value) by 3.2% each year. What would the constant ratio be?

4. Many items that you buy will decrease in value over time. For example; cars, computers, and cell phones usually decrease in value This is called ***depreciation***. A new car ***depreciates*** by approximately 12% each year.

You bought a new Jeep for $25 000. Complete the table below to determine the annual value of your new Jeep for the first five years that you own it.

Year	Starting Value	Value that the Depreciation is Calculated on	Depreciation Amount	Final Value
1	25 000			
2				
3				
4				
5				

6.2.2 Compound Interest (continued)

5. Kelly deposits $750 into a savings account that pays compound interest. The table below shows her annual balance of this investment. What interest rate did the bank give Kelly?

Year	Final Balance
1	795
2	842.70
3	893.26
4	946.86
5	1003.67

6. When you were born, your parents deposited $5000 into a bank account to pay for your college or university education. The bank account pays interest at a rate of 4% per year.
Complete the table below to determine how much money your parents will have for your education. Some entries have been completed for you to check your work.

Year	Starting Balance ($)	Balance that Interest is Calculate on ($)	Interest ($)	Ending Balance ($)
1	5000	5000	200	5200
2	5200	5200	208	5408
3				
4				
5			233.97	6083.26
6	6083.26	6083.26	243.33	6326.59
7	6326.59	6326.59		
8				6842.84
9	6842.84	6842.84	273.71	7116.55
10	7116.55	7116.55	284.66	7401.21
11	7401.21			7697.26
12	7697.26	7697.26	307.89	8005.15
13				
14	8325.36	8325.36	333.01	8658.37
15	8658.37	8658.37		
16				
17	9364.89	9364.89	374.60	9739.49

Unit 6 : Day 8 : Investigations with the TVM Solver

Grade 11 U/C

Minds On: 15

Action: 45

Consolidate:15

Total=75 min

Description/Learning Goals

- Review the use of the TVM solver to determine future value, present value, number of payments, etc. involving annuities
- Investigate the effect of changing payment amount, payment frequency, interest rate, compounding frequency on an annuity

Materials

- Class set of TI-83+ (or TI-84+)
- Viewscreen
- BLM 6.8.1
- BLM 6.8.2
- BLM 6.8.3
- BLM 6.8.4
- BLM 6.8.5

Assessment Opportunities

Minds On…

Whole Class → Review

Have students complete BLM 6.8.1 to review solving annuity problems with the TVM Solver.

You can use the viewscreen to display solutions.

Curriculum Expectations/Observation/Checklist: Assess students' ability to solve annuity problems with the TVM solver.

Teacher can use this time for an informal assessment

Action!

Small Groups → Investigation

Organize the class into groups of 4.

Groups investigate the effect of changing various parameters of annuities (payment amount, payment and compounding frequency, interest rate). All groups should complete BLM 6.8.2 – 6.8.4.

Mathematical Process Focus: Problem Solving / Connection (Students will **solve problems** involving annuities and investigate **connections** between the parameters of annuities

Consolidate Debrief

Whole Class → Reflection

Have groups communicate their findings/conclusions. The teacher can list 'Key Ideas' from each investigation, and call upon individual groups to add information from their summaries. (e.g. 'the future value of an annuity is proportional to the payment amount', or 'the number of payments needed to repay a loan increases with the interest rate, but not in a linear or exponential pattern')

Verify that the conclusions are accurate/complete and have students record the 'Key Ideas' in their notes

Literacy Strategy: You can use a 'Grafitti' approach, where groups add their own key ideas to chart paper placed around the room. See *Think Literacy . Cross-Curricular Approaches Grades 7-12. 2003. p. 66*

Home Activity or Further Classroom Consolidation

- Additional Practice – BLM 6.8.5

The additional practice may be used as an assessment piece.

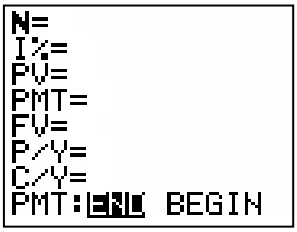
Application
Concept Practice

Curriculum Expectations/Worksheet/Marking Scheme: Assess students' ability to solve annuity problems with the TVM solver.

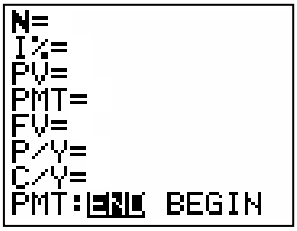
6.8.1 Practice with Annuities and the TVM Solver

Use the TVM Solver to solve each problem.

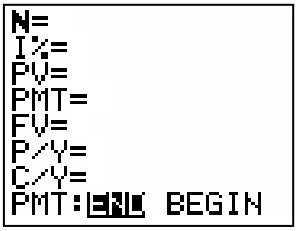
1. Determine the future value of 20 annual deposits of $1000 if the deposits earn 8% interest per annum, compounded annually.



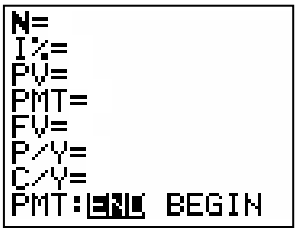
2. Determine the monthly payments required to accumulate a future value of $10 000 in four years, if the payments earn 6.5% interest per annum, compounded monthly.



3. A $10 000 loan is repaid with monthly payments of $334.54 for three years. Determine the interest rate per annum, compounded monthly.



4. Determine the total amount of interest earned on an annuity consisting of quarterly deposits of $1500.00 for ten years, if the annuity earns 9% interest per annum, compounded quarterly.



6.8.2 Investigating Changes in the Payment

1. In this exercise, you will investigate the effects of changing the payment amount on an ordinary annuity. Use the TVM Solver to complete the following chart.

Payment Amount	Payment Frequency	Compounding Frequency	Number of Payments	Interest Rate (per annum)	Future Value of Annuity
$100	Monthly	Monthly	120	8%	
$200	Monthly	Monthly	120	8%	
$300	Monthly	Monthly	120	8%	
$400	Monthly	Monthly	120	8%	
$500	Monthly	Monthly	120	8%	

a) Does the future value increase by the same amount each time? Explain.

b) What happens to the future value when the payment amount doubles?

c) What happens to the future value when the payment amount triples?

d) Summarize the effect of changing the payment amount on the future value of an annuity when all other conditions remain the same.

6.8.2 Investigating Changes in the Payment (Continued)

2. In this exercise, you will investigate how changes in the payment amount affect the length of time needed to repay of a loan. Use the TVM Solver to complete the following chart.

Loan Amount	Payment Amount	Payment Frequency	Compounding Frequency	Interest Rate (per annum)	Number of Payments
$10 000	$100	Monthly	Monthly	4%	
$10 000	$200	Monthly	Monthly	4%	
$10 000	$300	Monthly	Monthly	4%	
$10 000	$400	Monthly	Monthly	4%	
$10 000	$500	Monthly	Monthly	4%	

a) Does the number of payments change by the same amount each time? Explain.

b) What happens to the number of payments when the payment amount doubles?

c) What happens to number of payments when the payment amount triples?

d) Summarize the effect of changing the payment amount on the number of payments required to pay back a loan when all other conditions remain the same.

6.8.3 Investigating Changes in the Payment Frequency

1. In this exercise, you will investigate the effects of changing the payment and compounding frequency of an ordinary annuity. Use the TVM Solver to complete the following chart.

Payment Amount	Payment Frequency	Compounding Frequency	Number of Payments	Interest Rate (per annum)	Future Value of Annuity
$1200	Annually	Annually	10	9%	
$600	Semi-Annually	Semi-Annually	20	9%	
$300	Quarterly	Quarterly	40	9%	
$100	Monthly	Monthly	120	9%	
$23.08	Weekly	Weekly	520	9%	
$3.29	Daily	Daily	3650	9%	

a) Verify that the total of all payments made is the same for each case.

b) Does the future value increase by the same amount each time? Explain.

c) Which change in payment frequency results in the greatest change in future value?

d) Summarize the effect of changing the payment frequency on the future value of an annuity when all other conditions remain the same.

6.8.3 Investigating Changes in the Payment Frequency
(Continued)

2. In this exercise, you will investigate how changes in the payment frequency affect the length of time needed to repay of a loan. Use the TVM Solver to complete the following chart.

Loan Amount	Payment Amount	Payment Frequency	Compounding Frequency	Interest Rate (per annum)	Number of Payments
$10 000	$1200.00	Annually	Annually	11%	
$10 000	$600.00	Semi-Annually	Semi-Annually	11%	
$10 000	$300.00	Quarterly	Quarterly	11%	
$10 000	$100.00	Monthly	Monthly	11%	
$10 000	$23.08	Weekly	Weekly	11%	
$10 000	$3.29	Daily	Daily	11%	

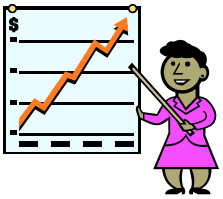
a) Which payment frequency pays off the loan in the least amount of time?

b) Determine the total amount of interest paid using annual payments.

c) Determine the total amount of interest paid using daily payments.

d) Summarize the effect of changing the payment frequency on the number of payments required to pay back a loan when all other conditions remain the same.

6.8.4 Investigating Changes in the Interest Rate



1. In this exercise, you will investigate the effects of changing the interest rate of an ordinary annuity. Use the TVM Solver to complete the following chart.

Payment Amount	Payment Frequency	Compounding Frequency	Number of Payments	Interest Rate (per annum)	Future Value of Annuity
$50	Weekly	Weekly	520	2%	
$50	Weekly	Weekly	520	4%	
$50	Weekly	Weekly	520	6%	
$50	Weekly	Weekly	520	8%	
$50	Weekly	Weekly	520	10%	
$50	Weekly	Weekly	520	12%	

a) Does the future value increase by the same amount each time? Explain.

b) Is there a common ratio between the future value amounts? Explain.

c) Summarize the effect of changing the interest rate of an annuity when all other conditions remain the same.

6.8.4 Investigating Changes in the Interest Rate (Continued)

2. In this exercise, you will investigate how changes in the interest rate affect the length of time needed to repay of a loan. Use the TVM Solver to complete the following chart.

Loan Amount	Payment Amount	Payment Frequency	Compounding Frequency	Interest Rate (per annum)	Number of Payments
$10 000	$200.00	Monthly	Monthly	2%	
$10 000	$200.00	Monthly	Monthly	4%	
$10 000	$200.00	Monthly	Monthly	6%	
$10 000	$200.00	Monthly	Monthly	8%	
$10 000	$200.00	Monthly	Monthly	10%	
$10 000	$200.00	Monthly	Monthly	12%	

a) Does the number of payments change by the same amount each time? Explain.

b) Determine the total amount of interest paid with an interest rate of 2% per annum.

c) Determine the total amount of interest paid with an interest rate of 12% per annum.

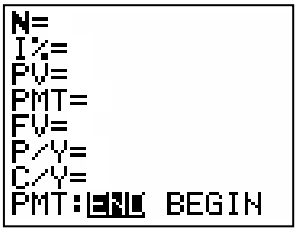
d) Summarize the effect of changing the interest rate on the number of payments required to pay back a loan when all other conditions remain the same.

6.8.5 Changing Conditions of an Annuity - Practice

Use the TVM Solver to solve each problem.

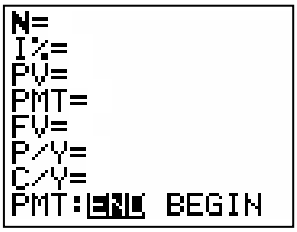
1. A student begins saving for college by making regular monthly payments of $200.00 into an account that earns 5% per annum interest, compounded monthly.

   a. Determine the value of the annuity after 4 years.

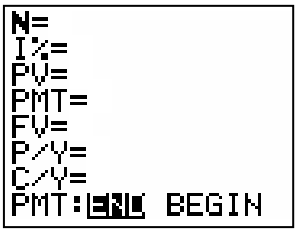


   b. If the amount of the payments was changed to $300.00, what would the future value after four years be? (HINT – How can you determine this <u>without</u> the TVM Solver?)

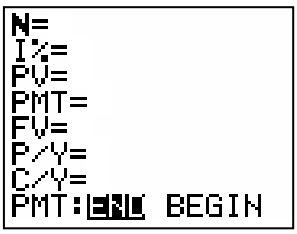
   c. Determine the amount of additional interest earned using $200.00 monthly payments with an interest rate of 8% per annum instead of 5% per annum.



2. a. Determine the monthly payments required to accumulate a future value of $10000.00 in four years, if the payments earn 6.5% interest per annum, compounded monthly.

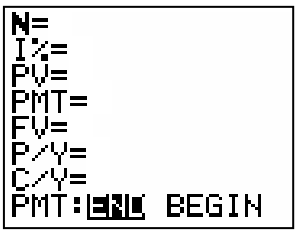


   b. What would the required weekly payments be to accumulate the same amount of money in the same amount of time? (Interest rate is 6.5% per annum, compounded weekly.)

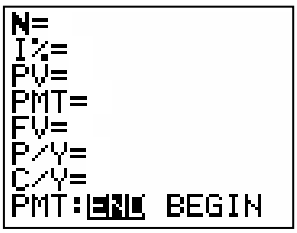


6.8.5 Changing Conditions of an Annuity - Practice (Continued)

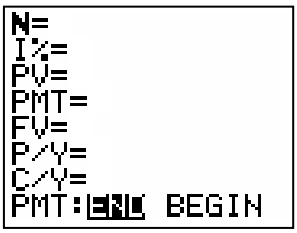
3. A $20 000 car loan is charged 3.9% per annum interest, compounded quarterly.
   a. Determine the quarterly payments needed to pay the loan off in five years.



   b. How much faster would the loan be paid off using the same payments, if the interest rate was lowered to 1.9%?



c. How much in interest charges could be saved (compared to part a.) by making weekly payments of $100.00, if interest is charged at 3.9%, compounded weekly.



4. Luke and Laura are each paying off loans of $5000.00. Luke makes monthly payments of $75.00 and interest charged at 9% per annum, compounded monthly. Laura pays the loan off in the same amount of time, but her monthly payments are only $65.00. Determine the annual interest rate that Laura is charged.